

# Household investments, limited participation and equity premium with wealth heterogeneity

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## Abstract

We develop and solve analytically a general equilibrium model where investors have heterogeneous wealth levels and exhibit uncertainty aversion. The model matches several salient patterns of household investments: (i) a sizeable fraction of households do not participate in the stock market, (ii) richer households are more likely to participate, and (iii) among the households that do participate, wealthier ones invest a larger share of their wealth into risky assets. These investment patterns determine the relation between the equity premium and market participation as a function of household wealth. For wealth growth that preserves or decreases wealth inequality, the resulting higher participation level is associated with a lower equity premium; whereas a rise in wealth inequality leads to the opposite relationship: lower participation at a lower equity premium.

*JEL Classifications:* D31, D51, D81, G11

*Keywords:* household portfolios; limited participation; equity premium; wealth heterogeneity; uncertainty aversion.

# 1 Introduction

Several patterns of household investment behavior are remarkably robust across countries and time periods: (i) a considerable fraction of households do not participate in the stock market; (ii) those who do tend to be wealthier; and (iii) among the households who participate, wealthier ones tend to invest a larger proportion of their wealth into risky assets.<sup>1</sup> These patterns have attracted considerable attention in recent research. However, most existing models focus on one or the other pattern independently rather than providing a joint explanation (as discussed in subsection 1.1). We develop a model that can match these empirical investment patterns in a *unified* way.

We also contribute to the strand of literature examining the asset pricing implications of limited participation. Empirical research has shown the importance of distinguishing between stockholders and non-stockholders when taking a model to the data.<sup>2</sup> Motivated by this research, a large body of theoretical work has emerged that formally explores how the degree of stock market participation affects key asset pricing variables, most notably the equity premium: Saito (1995), Basak and Cuoco (1998), Guo (2004), Polkovnichenko (2004), Cao, Wang, Zhang (2005), Guvenen (2009), Bach and Moller (2011), and Ui (2011). We contribute to this literature by showing that wealth inequality is an important factor for the relationship between participation choices and the equity premium.

The structure of our model is as follows. We consider an economy with uncertainty-averse agents endowed with different levels of wealth. The cross-section of household wealth follows a (truncated) Pareto distribution, which is consistent with the data. The agents can invest in a riskless bond and a risky asset interpreted as the stock market. Following Gilboa and

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<sup>1</sup>Empirical evidence regarding participation and its relation to wealth is provided in Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Bertaut (1998), Guiso, Haliassos, and Japelli (2003), Vissing-Jorgensen (2003), Campbell (2006), Calvet, Campbell, and Sodini (2007), Shum and Faig (2006), and Van Rooij, Lusardi, and Alessie (2011). The pattern of the proportion of risky investments increasing in wealth is documented by Bertaut and Starr-McCluer (2000), Perraudin and Sorensen (2000), Guiso, Haliassos, and Japelli (2002), and Vissing-Jorgensen (2002).

<sup>2</sup>See Mankiw and Zeldes (1991), Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), and Vissing-Jorgensen (2002), among others.

Schmeidler (1989), we model uncertainty aversion by assuming that investors entertain a set of possible distributions for the stock return, and base their investment decision on the worst-case distribution they consider possible.<sup>3</sup> To reflect the evidence that richer individuals tend to have better understanding of the stock market, we posit that the magnitude of an investor’s uncertainty about the stock market is inversely related to their wealth.

We find that one of two possible equilibria occurs, depending on how unevenly wealth is distributed in the economy: either the full participation equilibrium, in which all agents hold at least some amount of their wealth in the risky asset, or the limited participation equilibrium, in which a subset of agents optimally refrains from risky investment and only holds the risk-free bond. The limited participation equilibrium prevails at higher levels of wealth inequality.

In the full participation equilibrium, wealthier agents hold riskier portfolios. The equilibrium equity premium has two components, one capturing the premium for bearing fundamental risk of the stock—i.e. the variation in possible payoff realization—and one capturing the uncertainty premium—related to investors’ uncertainty about the true expected payoff underlying its distribution.

We then examine the limited participation equilibrium, which is more empirically relevant: Although participation rates have been increasing over the past decades, the 2007 Survey of Consumer Finance notes that only about half of the US population invests in the stock market, directly or indirectly through retirement plans or professional money management.<sup>4</sup> In this equilibrium, there exists a wealth threshold below which investors do not participate in the stock market, and above which they are long the stock. We find that wealthier market participants invest a larger fraction of wealth in the stock. Hence, our model is consistent with the three salient patterns of household investment decisions.

Next, we explore the relation between the degree of participation and the equity pre-

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<sup>3</sup>Another term for “uncertainty aversion” used in the literature is “ambiguity aversion.” We use these two terms interchangeably throughout the paper.

<sup>4</sup>Most other countries tend to be at even lower levels of stock market participation.

mium. This question has received much attention in the asset pricing literature since an influential paper by Basak and Cuoco (1998). By showing that a lower participation is associated with a higher equity premium, they make a point that limited participation can help to explain the equity premium puzzle.<sup>5</sup> In this paper, as well as in other studies obtaining a negative relation between participation and equity premium (Heaton and Lucas (2000), Polkovnichenko (2004)), Guvenen (2009)), the limited participation is assumed *exogenously*. The exogeneity assumption has turned out to be a pivotal one as recent models that *endogenize* the participation decision show that the reverse relation could well occur (Cao, Wang, and Zhang (2005), Ui (2011)). We contribute to this literature by examining, in a model with endogenous limited participation, the role of wealth inequality in determining how participation is related to the equity premium.

We examine how participation and equity premium are affected under three types of households' wealth growth—growth that preserves wealth inequality, increases it, and decreases it.<sup>6</sup> For growth that preserves or reduces wealth inequality, we find that a higher stock market participation is associated with a lower equity premium, consistent with exogenous participation models. If, instead, wealth inequality increases, stock market participation decreases, as does the equity premium. The intuition is as follows. On the one hand, a lower participation implies that fewer investors carry more of the risk of the stock market, which pushes the equity premium up. On the other hand, those who remain participants in the stock market are precisely the agents more willing to take risks, who are thus willing to accept a lower risk premium. The latter effect is not present in models with exogenous participation, whereas in our work it is present and can dominate the former effect when wealth inequality increases. A broad conclusion from this analysis is that when one seeks to understand how participation affects asset pricing, an important question to be asked is

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<sup>5</sup>The puzzle is that the standard asset pricing model generates a too low equity premium for plausible calibrations of the parameters, as discovered by Mehra and Prescott (1995).

<sup>6</sup>Research on wealth inequality finds that economic growth can have different effects on wealth inequality, depending on the country and time period considered (Atkinson (1997), Cagetti and De Nardi (2008)). Accordingly, we consider three types of wealth growth, and not one particular type, so as not to restrict the generality of our analysis.

what is the economic mechanism that causes changes in participation and asset prices, as different mechanisms can lead to different asset pricing implications.

## 1.1 Related literature

Our paper contributes to the literature investigating financial market implications of uncertainty aversion; examples are Dow and Werlang (1992), Chen and Epstein (2002), Epstein and Schneider (2003, 2007, 2008), Cao, Wang, and Zhang (2005), Garlappi, Uppal, and Wang (2007), Easley and O’Hara (2009), Cao, Han, Hirshleifer, and Zhang (2011), Ui (2011), and Pflug, Pichler, Wozabal (2012). Epstein and Schneider (2010) provide an excellent overview of recent developments in this area. While non-participation is a common result in these works, these studies do not explain why it is richer investors who are more likely to participate in the stock market.

It is worth noting that there are other approaches to modeling agents’ behavior in uncertain environments, which are conceptually related to, but in some respects different from, Gilboa and Schmeidler’s (1989) approach adopted in our paper. Important examples are the “smooth ambiguity” methodology of Klibanoff, Marinacci, and Mukerji (2005) and the approach relying on robust control theory (see Hansen and Sargent (2007) for a textbook treatment).

A number of papers examine the role of participation costs (along with other factors) in causing limited participation (Orosel (1998), Cocco (2005), Peress (2005), Gomes and Michaelides (2008), Favilukis (2012)). However, as noted by Easley and O’Hara (2009), “[w]hile undoubtedly an important factor for some investors, the empirical evidence suggests that this can only be a partial explanation.” A similar argument is made by Cochrane (2007): “The costs of joining the stock market are trivial...Thus, people who do not invest at all *choose* not to do so in the face of trivial fixed costs.” See Andersen and Nielsen (2011) for an empirical study demonstrating that participation costs are unlikely to explain why

many individuals do not invest in the stock market.

Another strand of literature explains non-participation by considering various nonstandard preferences, e.g., preferences capturing loss aversion (Gomes (2005), Barberis, Huang, and Thaler (2006)), disappointment aversion (Ang, Bekaert and Liu (2005)), and rank-dependent preferences (Polkovnichenko (2005), Chapman and Polkovnichenko (2009)). Other mechanisms leading to limited participation are explored in Hong, Kubik, and Stein (2004), Linnainmaa (2005), Guiso, Sapienza, and Zingales (2008), Berk and Walden (2012), and Gormley, Liu, and Zhou (2010).

The above literature is largely silent on why, among those investors who choose to participate, richer ones put larger shares of wealth in risky assets. This pattern is explained in Peress (2004), Roussanov (2010), and Wachter and Yogo (2010), but these works do not have limited participation. Peress (2005) explains jointly the three patterns of household investments, but in a setting with participation costs and asymmetric information. Moreover, he does not examine the relation between equity premium and the extent of participation.

On a broad level, because wealth heterogeneity plays a key role in our analysis, our paper contributes to the rapidly growing heterogeneous agent literature that establishes the value of going beyond the traditional representative agent paradigm to explain various empirical phenomena (see Hommes (2006) and Heathcote, Storesletten, and Violante (2009) for literature reviews).

The rest of the paper proceeds as follows. Section 2 presents the model and supporting evidence for its key assumptions. Section 3 characterizes the equilibrium under full and limited stock market participation. Section 4 explores how wealth inequality affects the degree of stock market participation and equity premium. Section 5 concludes. The Appendix contains all proofs.

## 2 Economic setting

### 2.1 Basic set-up

We consider an economy with two assets, a riskless bond and a risky stock, where the stock is taken to represent the entire stock market. The bond is in perfectly elastic supply and acts as a numeraire. The return on the bond is normalized to zero. The stock payoff per unit of stock, denoted by  $x$ , is normally distributed with mean  $\mu > 0$  and standard deviation  $\sigma > 0$ .

There is a continuum of investors, indexed by  $i \in [0, 1]$ , who are heterogeneous in their wealth. Without loss of generality, we assume that investors have no bond endowments, allowing us to use the terms “wealth” and “stock endowments” interchangeably throughout the paper. Denoting by  $e_i$  the stock endowment of investor  $i$ , we posit that

$$e_i = \frac{e_H}{1 + i(e_H - e_L)/e_L}. \quad e_H > e_L, \quad (1)$$

From (1), investor  $i = 0$  has the highest endowment  $e_H$ , and as  $i$  increases the endowment monotonically decreases, reaching the lowest value  $e_L$  for investor  $i = 1$ . This specification is convenient as it is both analytically tractable and consistent with the observed Pareto wealth distribution.<sup>7</sup> To see that (1) implies a (truncated) Pareto distribution,<sup>8</sup> consider the distribution function  $F(e)$ —for a given endowment  $e \in [e_L, e_H]$ ,  $F(e)$  is the fraction of investors whose endowment is below  $e$ . Since  $e_i$  monotonically decreases in  $i$ , we get that  $F(e)$  equals  $1 - i(e)$  where  $i(e)$  is the index of the investor whose endowment equals  $e$ . Thus, we have

$$F(e) = 1 - i(e) = \frac{e_H}{e_H - e_L} \left(1 - \frac{e_L}{e}\right), \quad (2)$$

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<sup>7</sup>Persky (1992) and, more recently, Gabaix (2009) provide reviews of the relevant evidence. Gabaix also discusses a number of other contexts in finance and economics in which Pareto distribution plays an important role.

<sup>8</sup>“Truncated” because in our model wealth is bounded from above by  $e_H$ , while a true Pareto distribution is unbounded.

which is a truncated Pareto distribution function.

Integrating over all agents' individual endowments (1) gives the average endowment in the economy  $\bar{e}$ :

$$\bar{e} = \int_0^1 e_i di = \int_0^1 \frac{e_H}{1 + i(e_H - e_L)/e_L} di = \frac{e_H e_L}{e_H - e_L} \ln \left( \frac{e_H}{e_L} \right). \quad (3)$$

There are two time periods: an initial trading period  $t = 0$  when price-taking investors form their portfolios, and a consumption period  $t = 1$  when the stock payoff is realized and investors consume their terminal wealth  $w_i$ . Denoting by  $\theta_i$  the share of initial wealth that investor  $i$  invests in the risky stock, his time-1 wealth is

$$w_i = (\theta_i x + (1 - \theta_i)p)e_i, \quad (4)$$

where  $p$  denotes the stock price at time 0.

## 2.2 Ambiguity aversion

We assume that investors know the true value of the stock's volatility  $\sigma$ , but are uncertain about the value of the expected return  $\mu$ . This assumption has become commonplace in the literature. Kogan and Wang (2003), Cao, Wang, and Zhang (2005), Garlappi, Uppal, and Wang (2007), Cao, Han, Hirshleifer, and Zhang (2011), among others, all consider settings in which investors are uncertain about stock mean returns rather than return volatilities.<sup>9</sup> The leading explanation is that obtaining a precise estimate of an asset's expected return is more difficult than doing the same for the asset's volatility (Merton (1980)). Investor  $i$  has multiple priors over the true value of mean stock payoff  $\mu$ , represented by an interval

$$\mathcal{M}_i = [\mu - u_i, \mu + u_i], \quad (5)$$

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<sup>9</sup> Exceptions are recent papers by Faria, Correia-da-Silva, and Ribeiro (2010) and Epstein and Ji (2011) which model ambiguity about volatility.



where the interval size parameter  $u_i$  is agent-specific, as described in subsection 2.3. One interpretation of this interval is that, when estimating the expected stock return from past data, investors recognize that the true value of  $\mu$  is likely to be different from their point estimate. Therefore, when making his investment decision, investor  $i$  entertains a set of possible values for  $\mu$ , and the size of this set,  $2u_i$ , measures his degree of uncertainty about the estimate.<sup>10</sup>

Investors are averse to both the fundamental risk posed by the volatility of the stock's payoff,  $\sigma > 0$ , as well as to the uncertainty regarding the true level of expected payoff  $\mu$ . The former is accounted for by the utility function's concavity, whereas to capture ambiguity aversion we follow Gilboa and Schmeidler (1989)—we assume that investors maximize utility conditioning on their perceived worst-case scenario for the stock mean return. Formally, investor  $i$  solves the max-min problem

$$\max_{\theta_i} \min_{\tilde{\mu} \in \mathcal{M}_i} E[-\exp(-\alpha_i w_i)], \quad (6)$$

where wealth  $w_i$  is given in equation (4), and  $\alpha_i$  is investor  $i$ 's absolute risk aversion parameter, as specified in subsection 2.3.

### 2.3 Modeling risk aversion and degree of ambiguity

Empirically, many types of heterogeneity across investors (or potential investors) correlate with wealth. In contrast to existing models with ambiguity aversion—where wealth, absolute risk aversion, and degree of uncertainty are typically treated as independent parameters—our model relates these agent-specific model parameters to be in line with empirical evidence.

Wealthy individuals tend to exhibit lower values of absolute risk aversion (ARA) than poor individuals, as documented in Wolf and Pohlman (1983), Saha, Shumway, and Talpaz

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<sup>10</sup>Kogan and Wang (2003) and Garlappi, Uppal, and Wang (2007), among others, also interpret an investor's uncertainty interval as representing his confidence interval around the point estimate of the mean stock return.

(1994), Guiso and Paiella (2004, 2008), and Paravisini, Rappoport, and Ravina (2011). To capture this evidence and at the same time retain the tractability, we assume constant absolute risk aversion (CARA) preferences where the ARA parameter is investor-specific—wealthier investors have lower ARA.<sup>11</sup> Specifically, we assume

$$\alpha_i = \frac{\gamma}{e_i}, \quad \gamma > 0. \quad (7)$$

The functional form (7) is chosen so as to emulate the widely used constant relative risk aversion utility for which ARA is equal to the ratio of relative risk aversion to wealth. Accordingly, parameter  $\gamma$  in (7) can be viewed as reflecting relative risk aversion.

Another characteristic of investors that is related to wealth is the knowledge about the stock market. Wealthier investors tend to pay more attention to, and have a better understanding of, financial markets than poorer ones. Lewellen, Lease, and Schlarbaum (1977) find that richer investors consume more financial periodicals and investment research services. Bernheim (1998) documents that investors’ financial scores increase with earnings. The survey of Donkers and van Soest (1999) indicates that a person’s interest in financial matters increases with income. Guiso and Jappelli (2005) find a positive relation between wealth and financial awareness, while Guiso and Paiella (2009) document the same positive relationship between wealth and financial literacy.

Accordingly, we assume that wealthier investors, i.e., the ones with higher stock endowments, have lower uncertainty about the expected payoff  $\mu$ . In particular, we posit that the relation between investor  $i$ ’s degree of uncertainty  $u_i$ —which determines the size of the

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<sup>11</sup> A similar approach to modelling ARA parameter is used in Baker and Hall (2004), Peress (2005), Makarov and Schornick (2010), and He (2011). An alternative way of accounting for decreasing ARA, which does not require investor heterogeneity, is to assume that agents have the standard power utility function (CRRA). We do not follow this route because the resulting model cannot be solved analytically. Adopting a CARA-normal framework (or, relatedly, a mean-variance framework) is typical for uncertainty aversion models (see, e.g., Cao, Wang, and Zhang (2005), Garlappi, Uppal, and Wang (2007), and Easley and O’Hara (2009)).

interval  $\mathcal{M}_i$  as given in (5) - and his endowment  $e_i$  is

$$u_i = \frac{1}{ae_i}. \tag{8}$$

Parameter  $a$  in (8) can be viewed as a proxy for the level of information technologies such as television, internet, etc., that allow (potential) investors to gather information. When  $a$  is higher, investors can more easily access information about the stock market, implying a lower uncertainty for a given level of wealth. Henceforth, we refer to  $a$  as information accessibility.

## 2.4 Discussion of modeling choices

Our goal is to develop a parsimonious model that can be solved analytically, which necessitates simplifying assumptions. We now comment on several such assumptions and mention possible generalizations.

We adopt a two-period setting because it permits analytical solutions while at the same time allowing us to examine the *cross-sectional* differences in household investments, which is our focus. Considering a full-fledged dynamic model is not likely to qualitatively affect our cross-sectional predictions, but the resulting model will be more involved. In considering two assets, one risky and one riskless, we focus on the overall risk of household portfolios, as opposed to portfolio compositions. Obviously, explaining some other patterns of household investments, e.g. life-cycle patterns or underdiversification, would require a model with multiple periods and/or multiple risky assets. Addressing these issues is beyond the scope of the current paper.

In our model, investors with lower wealth unequivocally have higher uncertainty about the stock return than wealthier ones, as seen from specification (8). In reality, this negative relation between wealth and uncertainty is not likely to be perfectly monotonic—*some* individual wealthier investors could well be more uncertain than *some* poorer investors. This

feature could be incorporated into our model by introducing a noise term into investors' uncertainty intervals, so that (5) would be replaced by a more general specification

$$\mathcal{M}_i = [\mu - u_i - \epsilon_i, \mu + u_i + \epsilon_i],$$

where  $\epsilon_i$  is a random noise term capturing other factors, beyond wealth, that may affect his degree of ambiguity. This modification would complicate the analysis but would not affect our main predictions as long as the noise terms are not systematically lower for poorer investors.

One can also generalize our simple relation between wealth and uncertainty, given in (8), and consider a more general specification  $u_i = (ae^i + c)^{-\phi}$ , of which (8) is a special case for  $c = 0$  and  $\phi = 1$ . Parameter  $\phi > 0$  captures the elasticity of the level of uncertainty with respect to wealth, whereas parameter  $c > 0$  ensures that the degree of uncertainty  $u_i$  remains bounded from above for low endowment values  $e_i > 0$ . This generalization would not affect our main findings.

### 3 Equilibrium

The definition of an equilibrium in our economy is standard: Each investor  $i$  takes the stock price  $p$  as given and chooses his investment  $\theta_i$  in order to maximize the worst-case expected utility. The equilibrium stock price  $p$  is such that the total demand and supply for the stock are equal.<sup>12</sup>

**Definition 1** *An equilibrium is given by a stock price  $p$  and investors' portfolio weights  $\theta_i, i \in [0, 1]$  in the stock, such that:*

*(i) given  $p$ , each  $\theta_i, i \in [0, 1]$ , solves investor  $i$ 's optimization problem (6),*

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<sup>12</sup>In subsection 2.1, we used  $p$  and  $\theta_i$  to denote generically the stock price and stock weights, respectively. To avoid excessive notation, hereafter we use the same symbols to denote the corresponding equilibrium.

(ii) the total number of stocks demanded by the investors,  $\int_0^1 \theta_i e_i di$ , equals the stock supply  $\int_0^1 e_i di$ .

As formally established in the Appendix (see proof of Proposition 1), there can be only two types of equilibrium, depending on the values of the parameters that characterize the economy. The full participation equilibrium, in which all investors invest a positive amount in the stock ( $\theta_i > 0$  for all  $i \in [0, 1]$ ), and the limited participation equilibrium, in which some investors have no stock holdings ( $\theta_i = 0$ ) and the remaining investors have a long position in the stock ( $\theta_i > 0$ ).

### 3.1 Equilibrium with full participation

We first examine the full participation equilibrium because we obtain explicit expressions for all equilibrium quantities, which facilitates a discussion of the underlying economic mechanisms. In the next subsection we explore the more empirically relevant limited participation equilibrium.

Proposition 1 presents the conditions under which the full participation equilibrium occurs, and reports the associated stock weights and the equity premium.

**Proposition 1** *An equilibrium with full participation occurs when*

$$\frac{e_H \ln(e_H/e_L) - e_H + e_L}{e_H e_L \ln(e_H/e_L)} < a\gamma\sigma^2. \quad (9)$$

*The equilibrium investment choice of investor  $i$ ,  $i \in [0, 1]$ , is*

$$\theta_i = 1 + \frac{1/\bar{e} - 1/e_i}{a\gamma\sigma^2}. \quad (10)$$

*Consequently,  $\partial\theta_i/\partial e_i = 1/(e_i^2 a\gamma\sigma^2) > 0$ , implying that wealthier investors allocate a higher*

*fraction of wealth to the risky asset.*

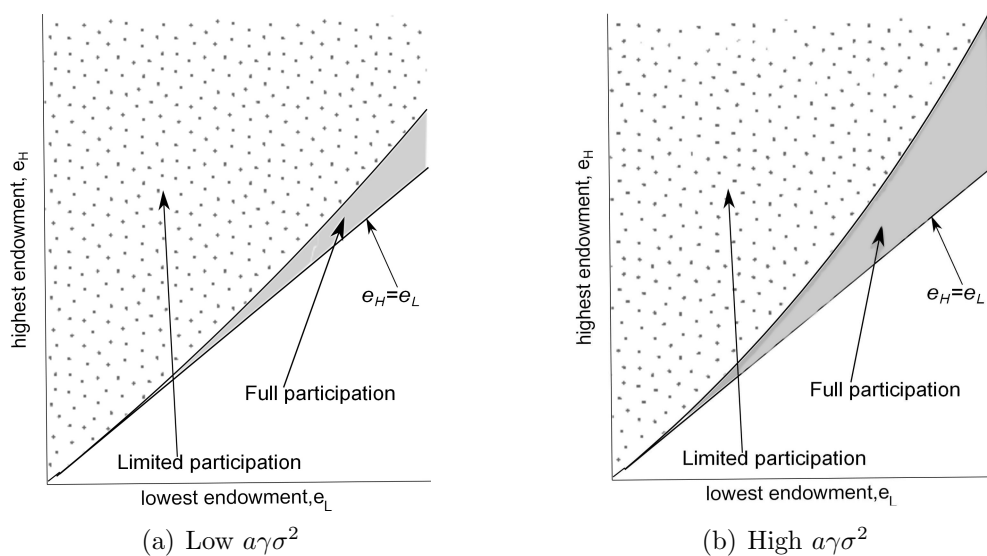
*The equilibrium equity premium is*

$$\mu - p = \gamma\sigma^2 + \frac{1}{a\bar{e}}. \quad (11)$$

To visualize the parameter conditions for the full participation equilibrium (9), Figure 1 depicts the region of parameters  $e_L$  and  $e_H$  in which this condition is satisfied. In both panels (a) and (b), full participation occurs in shaded areas, whereas limited participation occurs in dotted areas. All investors choose to participate in the stock market when the wealth gap between rich and poor investors is sufficiently narrow, i.e. wealth inequality in the economy is relatively low. The intuition is as follows. Consider first a limiting case when all investors have the same wealth,  $e_L = e_H$ . Given that these identical investors have identical stock holdings, in equilibrium either everyone participates or no one does. Since the stock is in positive net supply, the former must hold—the stock price adjusts until investors are willing to hold the stock. When wealth is heterogeneous but the heterogeneity is relatively small, the same full participation outcome obtains.

At large degrees of heterogeneity, the higher demand for risky assets from wealthy investors drives up prices sufficiently to make them unappealing to more conservative, i.e. less wealthy, investors, who then choose to opt out of owning risky stocks.

Another implication of Figure 1 is that, conditional on wealth distribution, the full participation equilibrium is more likely to occur when the quantity  $a\gamma\sigma^2$  is higher—the shaded area is larger in panel (b) than in panel (a). A higher information accessibility  $a$  benefits all investors by reducing their uncertainty, but more so the relatively poor investors. Hence, these poor investors are more like to invest in the stock market when  $a$  increases, and so full participation is more likely. An increase in either risk aversion  $\gamma$  or stock volatility  $\sigma$  makes the stock less attractive, which pushes down the equilibrium stock price. This lower price also makes the stock more attractive for the more uncertainty averse investors, and full



**Figure 1: Full and limited participation equilibrium.** The shaded area corresponds to the pairs of the lowest and highest endowment  $(e_L, e_H)$  for which the full participation equilibrium obtains, and the dotted area corresponds to the pairs  $(e_L, e_H)$  for which the limited participation equilibrium obtains. As we go from panel (a) to panel (b), the quantity  $a\gamma\sigma^2$  increases, where  $a$  is information accessibility,  $\gamma$  is the relative risk aversion parameter, and  $\sigma$  is the stock volatility. The plots are typical.

participation is more likely.

From equation (10) in Proposition 1, we see that the fraction of wealth an investor puts in the risky stock,  $\theta_i$ , increases with wealth  $e_i$ . Absent uncertainty aversion, the investors' stock weights  $\theta_i$ ,  $i \in [0, 1]$  would be independent of their initial wealth  $e_i$ , a standard result in settings with CRRA investors. In our model, wealthier investors have lower uncertainty about the risky stock than poorer ones, and so they find the stock more attractive. As a result, wealthier investors put a larger share of wealth in the stock.

The equity premium under full participation is a sum of two components that reflect investors' aversion to risk and uncertainty. The first component in equation (11) captures the risk premium arising due to the volatility of the stock's payoff. Analogously to otherwise similar models without uncertainty aversion, this component is proportional to investors' risk aversion  $\gamma$  and the stock volatility  $\sigma$ . The second term in (11) corresponds to the uncertainty premium. Because each investor's endowment determines his level of uncertainty, the

average endowment in the economy reflects the average uncertainty.<sup>13</sup> A higher information accessibility  $a$  lowers overall uncertainty in the economy and thus lowers the uncertainty premium.

### 3.2 Equilibrium with limited participation

In this subsection, we examine the limited participation equilibrium, characterizing the degree of stock market participation, the stock holdings of participating investors, and the equity premium. We show that our model is able to generate several salient household investment patterns observed empirically.

**Proposition 2** *An equilibrium with limited participation occurs when the condition for full participation (9) is not satisfied.*

*There exists a threshold level of wealth  $e^* \in (e_L, e_H)$  such that investors whose wealth is above  $e^*$  participate in the stock market, while investors whose wealth is below  $e^*$  do not participate.*

*The threshold  $e^*$  is implicitly given by*

$$\frac{e_H}{e^*} \ln \frac{e_H}{e^*} - \frac{e_H}{e^*} = a\gamma\sigma^2 e_H \ln \frac{e_H}{e_L} - 1. \quad (12)$$

*For all participants—investors whose wealth is  $e_i > e^*$ , the stock weight is*

$$\theta_i = \frac{1/e^* - 1/e_i}{a\gamma\sigma^2}, \quad \text{if } e_i > e^*, \quad (13)$$

*where  $e^*$  is given by (12). For investors whose wealth is  $e_i \leq e^*$ , the stock weight is  $\theta_i = 0$ .*

*Consequently, among the participants, richer individuals put a higher fraction of their wealth into the stock market than poorer investors in equilibrium.*

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<sup>13</sup>A similar decomposition of the equity premium into the risk and uncertainty premia is also obtained in other models with uncertainty aversion, e.g. Chen and Epstein (2002).



The equilibrium equity premium is

$$\mu - p = \frac{1}{ae^*}, \quad (14)$$

where  $e^*$  is given by (12).

As shown in Figure 1, limited participation occurs when wealth heterogeneity among investors is substantial enough, i.e., when the highest endowment  $e_H$  is sufficiently higher than the lowest endowment  $e_L$ . Given that wealth is related to lower levels of risk aversion and ambiguity (as seen from (7) and (8)), high wealth heterogeneity translates into high heterogeneity in incentives to invest in the stock, with the wealthier investors being considerably more aggressive in the amount of risk they take on in their investments. The resulting higher price due to the wealthier investors' demand compels the poorer ones to refrain entirely from investing in the stock. Among stock market participants, a higher wealth level  $e_i$  is associated with a larger the wealth share  $\theta_i$  invested in the stock. This result, and its intuition, is consistent with the full participation equilibrium.

As a final point, we would like to note the following implication of the above analysis for asset pricing models. In much of this work, the focus is on matching *aggregate* quantities such as stock prices, volatilities, correlations, etc., whereas the model-implied behavior of *individual* investors does not typically match the observed behavior. Our analysis specifically focuses on the latter and clarifies the economic mechanisms that, if introduced in a model, would work towards generating (i) non-participation among relatively poor investors, and (ii) stock weights that increase in wealth for investors who choose to hold risky stocks.

## 4 Equity premium and the degree of participation

Using the results on the limited participation equilibrium discussed above as a starting point, we now address a recurring question in the asset pricing literature: How the degree of stock market participation is related to the equity premium. Unlike existing work exploring this

question, our model accounts for the observed wealth heterogeneity, enabling us to investigate this question from a novel perspective. In particular, we will show that this heterogeneity—and the ensuing heterogeneity in decision making—is an important determinant of the relation between participation and equity premium.

Over the past decades, household wealth has been increasing due to economic growth, whereas wealth inequality has increased, decreased, or remained stable depending on the time period, country, and methodology being used. For example, looking at US data, wealth inequality exhibited a downward trend between 1920s and 1970s and an upward trend after that (Cagetti and De Nardi (2008)). Accordingly, we consider three scenarios of wealth growth—with different changes in the degree of inequality in the economy—to assess how participation as well as the equity premium are affected. As a measure of inequality we use the standard Gini index  $G$ , which is a number between zero and one; the higher  $G$  is, the higher is wealth inequality (formally,  $G$  is defined in equation (A16) in the Appendix).

**Definition 2** *We consider wealth growth of type 1, 2, and 3, defined by varying the parameters of the initial wealth distribution (1):*

type 1 – both  $e_L$  and  $e_H$  increase by the same factor.

type 2 –  $e_H$  increases,  $e_L$  does not change.

type 3 –  $e_L$  increases,  $e_H$  does not change.

For all three types of wealth growth characterized in Definition 2, each individual investor—not just the average—becomes richer, as follows from the fact that  $e(i)$  increases in  $e_L$  and  $e_H$  for any  $i \in [0, 1]$  (see equation (1)). The difference is which individuals benefit disproportionately from the overall growth in wealth. To measure how a given parameter of interest  $X$  (the Gini index, stock market participation, or the equity premium) is affected by each of the three types of growth, we compute three functional derivatives, denoted  $\Delta^{\text{type } n}[X]$ ,  $n = 1, 2, 3$ , that capture the impact of changing inequality on  $X$ .<sup>14</sup>

<sup>14</sup>Formal definitions are provided in equations (A12)–(A14) in the Appendix.

Lemma 1 computes the Gini index  $G$  for our economy, and reports how  $G$  is affected by the three types of wealth growth from Definition (2): type 1 wealth growth corresponds to the growth benefiting all investors equally, type 2 captures a scenario in which most of the growth in wealth accrues to the already wealthy agents, and type 3 favors poor investors.

**Lemma 1** *The Gini index  $G$  is*

$$G = 1 - 2 \frac{(e_H - e_L)(\ln e_H + 1) + e_L \ln e_L - e_H \ln e_H}{(e_H - e_L) \ln(e_H/e_L)}. \quad (15)$$

*The signs of the three functional derivatives are:  $\Delta^{type1}[G] = 0$ ,  $\Delta^{type2}[G] > 0$ , and  $\Delta^{type3}[G] < 0$ .*

*Consequently, type 1 wealth growth keeps wealth inequality ( $G$ ) in the economy constant, growth of type 2 raises inequality, and growth of type 3 lowers it.*

Proposition 3 reports how the degree of stock market participation and the equity premium are affected by wealth growth under the three different scenarios of inequality changes.

**Proposition 3** *The fraction of investors participating in the stock market  $i^*$  is indirectly given by*

$$(i^* + e_L/(e_H - e_L)) \ln(1 + i^*(e_H - e_L)/e_L) - i^* = \bar{e}a\gamma\sigma^2. \quad (16)$$

*Considering the three types of wealth growth given in Definition 2, the level of stock market participation increases when wealth inequality does not change (type 1) or decreases (type 3), but participation decreases when wealth inequality rises (type 2):*

$$\Delta^{type1}[i^*] > 0, \Delta^{type2}[i^*] < 0, \Delta^{type3}[i^*] > 0. \quad (17)$$

*The equity premium falls in response to a growth of aggregate wealth, regardless of its impact on inequality:*

$$\Delta^{type i}[\mu - p] < 0, \quad i = 1, 2, 3. \quad (18)$$

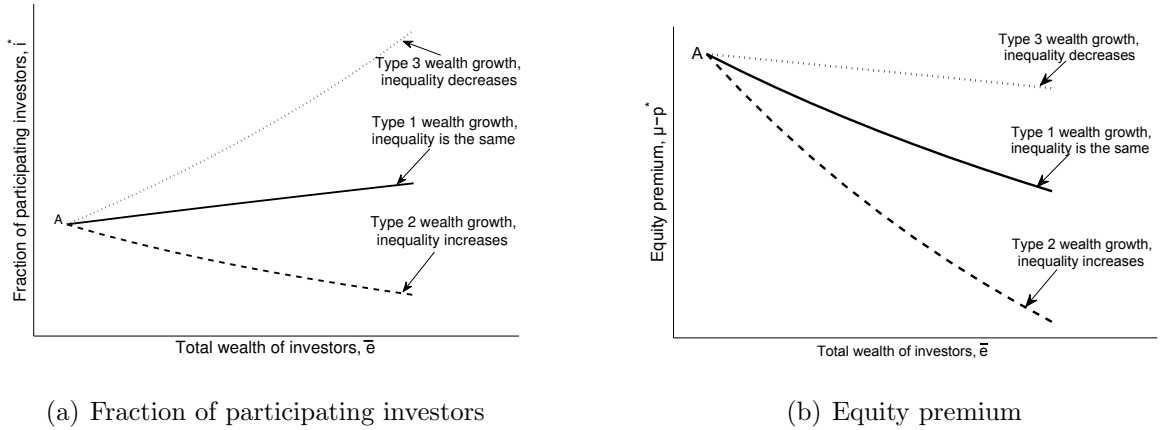
When information accessibility improves— $a$  increases—the equity premium decreases while participation increases:

$$\frac{\partial(\mu - p)}{\partial a} < 0, \quad \frac{\partial i^*}{\partial a} > 0. \quad (19)$$

For brevity, Proposition 3 presents only the signs of the sensitivities of equity premium  $\mu - p$  and degree of participation  $i^*$ , but it is also of interest to compare the magnitudes of these sensitivities. We plot these results in Figure 2. Panel (a) shows how the level of participation reacts to wealth growth under the three different scenarios of inequality changes. Panel (b) depicts the reaction of the equity premium in the three scenarios. The steepness of the dotted line in panel (a) reveals that the degree of participation increases faster under inequality-decreasing type-3 wealth growth than under inequality-preserving type-1 growth. The downward sloping dashed line reflects that inequality-increasing type-2 growth will tend to reduce participation.

To understand the intuition, note that there are two economic mechanisms affecting participation. First, as investors' endowments grow, the level of uncertainty each investor faces—and thus the overall level of uncertainty in the economy—decreases, which tends to increase participation. Second, lower wealth heterogeneity encourages higher participation, as established in Proposition 1. For type-1 growth that preserves inequality, only the first mechanism is at work. However, for type-3 growth, where inequality is reduced, both mechanisms work in the same direction, towards increasing participation. This also explains a faster rise in participation under type-3 growth relative to type-1. In contrast, For type-2 growth, when inequality rises, the two mechanisms work in the opposite directions. The second mechanism—driven by increasing wealth inequality—dominates the first, resulting in declining participation.

Panel (b) in Figure 2 shows the equity premium declining in response to a growth in wealth, regardless how this growth affects inequality. The higher wealth reduces overall uncertainty, lowering the uncertainty component of the equity premium which is thus unam-



**Figure 2: Effect of wealth growth on participation and equity premium.** In both panels (a) and (b), starting from point  $A$  we increase the total investors’ wealth allowing for varying impact on inequality, as defined in types 13 in Definition 2. Panel (a) depicts the effects on the equilibrium level of stock market participation  $i^*$ , and panel (b) on the equity premium  $\mu - p^*$ . The solid line corresponds to type 1 wealth growth which leaves wealth inequality unchanged, the dashed line to type 2 wealth growth which increases wealth inequality, and the dotted line to type 3 wealth growth which decreases wealth inequality. The plots are typical.

biguously lowered. To understand the relative positions of the three curves in the plot, note that in our model, an increase in participation always means that the “new” investors joining the stock market are poorer than the “old” participants. These poorer investors generally require a higher equity premium than richer ones because they are more risk averse and face more ambiguity; hence the equity premium is positively related to participation, keeping other things the same. Given this link between the equity premium and participation, the order of plots in panel (b) is the same as that in panel (a).

The last parameter discussed in Proposition 3 is information accessibility  $a$ . Better information accessibility reduces the overall uncertainty in the economy without affecting wealth heterogeneity, and this leads to a higher participation and a lower equity premium.

Combining the results of panels (a) and (b) in Figure 2 shows that the relation between participation and the equity premium can be either negative or positive, depending on the type of wealth growth and changes to information accessibility. For wealth growth that

decreases or preserves inequality (types 1 and 3), the relation is negative—a higher participation is associated with a lower equity premium, analogous to the implications found in models of exogenously limited participation. However, for type 2 growth, in which the rise in wealth is accompanied by increasing inequality, the relation is positive.

Keeping in mind the usual caveat that a two-period model like ours has limited ability to speak to long run trends, it can still be useful to look at Figure 2 from this angle. In particular, because wealth grows over time, we can broadly interpret the x-axis as corresponding to the passage of time. All three plots in panel (b) are consistent with the evidence that the equity premium has been steadily declining over time (Blanchard, Shiller and Siegel (1993), Fama and French (2002), Jagannathan, McGrattan and Scherbina (2000)). At the same time, the fraction of the population participating in the stock market has seen an upward trend (Bertaut and Starr-McCluer (2000) and Mankiw and Zeldes (1991)). To generate this pattern within our model, either wealth inequality needs to be stable or decreasing, or if the inequality is increasing the impact of improved information accessibility needs to be sufficiently strong to dominate the detrimental effect of higher wealth inequality on stock market participation.

Finally, our finding that accounting for endogenous participation leads to novel insights relative to studies with exogenous participation can be related to the following more general methodological question: when is it appropriate to model a variable as exogenous when in reality it is determined endogenously through agents' choices? If the economic link between this variable and agents' choices does not directly affect other model elements of interest, say asset prices, then treating the variable as exogenous is likely to be appropriate. If, however, the variable in question and asset prices are *both* directly affected, then relying on the exogeneity may lead to an incomplete and potentially distorted description of how the variable and asset prices are related. In our model, how wealth is distributed across investors directly affects both the degree of participation and the equity premium. This is why we uncover that, unlike the conclusion of exogenous participation models, limited participation

can be associated with a lower equity premium.

## 5 Conclusion

We develop a tractable general equilibrium model whose key features are wealth heterogeneity and uncertainty aversion. The model explains in a unified way the three salient features of household stockholding: (i) a sizeable fraction of households do not participate in the stock market, (ii) wealthier households are more likely to participate, and (iii) among the participants, wealthier households tend to invest a larger share of wealth into the stock market. We investigate how the degree of stock market participation affects the equity premium, in particular how the factor of wealth inequality is critical for determining whether higher participation is associated with a higher or lower equilibrium equity premia. For wealth growth that preserves or decreases wealth inequality, higher participation is associated with a lower equity premium, whereas the opposite relationship obtains when wealth inequality increases.

Our findings have implications for a growing body of research on the effects of limited participation on key financial and macroeconomic variables. Our analysis reveals that there is an important interplay between wealth heterogeneity and limited participation, and it can well be that a similar or richer interplay is present in other economic environments.

## A Appendix

**Proof of Proposition 1.** First, taking the stock price  $p$  as given, we solve for investors' optimal stock weights  $\theta_i$ ,  $i \in [0, 1]$ . Given CARA utility and normally distributed stock return, the original max-min problem (6) is equivalent to

$$\max_{\theta_i} \min_{\tilde{\mu} \in [\mu - u_i, \mu + u_i]} E[w_i] - \frac{\alpha_i}{2} \text{Var}[w_i]. \quad (\text{A1})$$

Computing the mean and variance of wealth (4), we get (keeping only terms that depend on  $\theta_i$ )

$$E[w_i] = \theta_i e_i (\mu - p), \quad \text{Var}[w_i] = \theta_i^2 e_i^2 \sigma^2. \quad (\text{A2})$$

Plugging (A2) and (7) in (A1) yields

$$\max_{\theta_i} \min_{\tilde{\mu} \in [\mu - u_i, \mu + u_i]} \theta_i (\mu - p) - \frac{\gamma \sigma^2}{2} \theta_i^2. \quad (\text{A3})$$

Solving the inner minimization problem in (A3) and using equation (8), we get that (A3) is equivalent to

$$\max_{\theta_i} \theta_i \left( \mu - \frac{\text{sgn}(\theta_i)}{ae_i} - p \right) - \frac{\gamma \sigma^2}{2} \theta_i^2. \quad (\text{A4})$$

The solution of (A4) can take one of the three forms, depending on whether the equity premium  $\mu - p$  belongs to  $(-\infty, -\frac{1}{ae_i})$ ,  $[-\frac{1}{ae_i}, \frac{1}{ae_i}]$ , or  $[\frac{1}{ae_i}, +\infty]$ . Solving (A4) in each of the three regions yields investor  $i$ 's optimal stock weight

$$\theta_i = \begin{cases} \frac{\mu - p + 1/(ae_i)}{\gamma \sigma^2} & \text{if } \mu - p < -1/(ce_i) & (\text{A5}) \\ 0 & \text{if } \mu - p \in [-1/(ae_i), 1/(ae_i)] & (\text{A6}) \\ \frac{\mu - p - 1/(ae_i)}{\gamma \sigma^2} & \text{if } \mu - p > 1/(ae_i). & (\text{A7}) \end{cases}$$

From (A5), we see that a necessary condition for some investor  $i \in [0, 1]$  to short the



stock is  $\mu - p < 0$ . In this case, from (A7), we see that *no* investor would want to long the stock, implying that the aggregate stock demand would be negative, while the aggregate stock supply (3) is positive. Hence, there cannot be an equilibrium when shorting takes place.

In the full participation equilibrium, investor  $i$ 's optimal portfolio is given by (A7) for all  $i \in [0, 1]$ . Multiplying (A7) by  $e_i$  yields the stock demand measured in stock units, integrating which across all investors we get the total number of stocks demanded by the investors:

$$\begin{aligned} \int_0^1 \frac{(\mu - p)e_i - 1/a}{\gamma\sigma^2} di &= \frac{\mu - p}{\gamma\sigma^2} \int_0^1 \frac{e_H}{1 + i(e_H - e_L)/e_L} di - \frac{1}{a\gamma\sigma^2} \\ &= \frac{\mu - p}{\gamma\sigma^2} \frac{e_H e_L}{(e_H - e_L)} \ln(e_H/e_L) - \frac{1}{a\gamma\sigma^2} = \frac{\mu - p}{\gamma\sigma^2} \bar{e} - \frac{1}{a\gamma\sigma^2}, \end{aligned} \quad (\text{A8})$$

where in the last equality we use equation (3). Equating the total demands and supply, (A8) and (3), respectively, yields (11).

The expression for equilibrium portfolios (10) is obtained by substituting the equilibrium equity premium (11) into (A7). Finally, the full participation condition (9) is obtained by noting from (A7) that every investor participates if  $\mu - p > 1/(ae_i)$  for all  $i \in [0, 1]$ , or, equivalently, that the poorest investor chooses to participate,  $\mu - p > 1/(ae_L)$ . Plugging in this participation condition the equity premium under full participation (11) and rearranging yields (9).

*Q.E.D.*

**Proof of Proposition 2.** Inspecting conditions (A6) and (A7), we see that the only possible structure of the limited participation equilibrium is as follows. Taking the equity premium  $\mu - p$  as given, we compute the wealth threshold  $e^*$  from

$$\mu - p = \frac{1}{ae^*}, \quad (\text{A9})$$

Then, investors whose endowment is above (below)  $e^*$  participate (do not participate) in the stock market. Differentiating distribution function (2), we get the density function of wealth

$$\frac{e_H e_L}{e_H - e_L} \frac{1}{e^2}.$$

Using this expression, we compute the aggregate demand for the risky stock (in number of units) by integrating the individual stock demands (A7) of investors richer than  $e^*$ :

$$\int_{e^*}^{e_H} \frac{(\mu - p)e - 1/a}{\gamma\sigma^2} \frac{e_H e_L}{e_H - e_L} \frac{1}{e^2} de = \frac{e_H e_L}{(e_H - e_L)\gamma\sigma^2} \left( (\mu - p) \ln \frac{e_H}{e^*} + \frac{1}{ae_H} - \frac{1}{ae^*} \right). \quad (\text{A10})$$

Equating (A10) to the aggregate supply (3), we get after simple algebra

$$a(\mu - p) \ln \frac{e_H}{e^*} - \frac{e_H}{e^*} = a\gamma\sigma^2 e_H \ln \frac{e_H}{e_L} - 1. \quad (\text{A11})$$

Plugging the equilibrium equity premium given in (A9) into (A11) yields (12). Finally, the equilibrium portfolios of stock market participants (13) is obtained by substituting the equity premium (A9) into (A7). *Q.E.D.*

**Proof of Lemma 1.** First, we formally define the functional derivatives  $\Delta^{\text{type } i}$ ,  $i = 1, 2, 3$ , for the three types of wealth growth presented in Definition 1. Denoting by  $X(e_L, e_H)$  the value of some parameter  $X$  under the initial wealth distribution (1) and by  $X(ke_L, ke_H)$  its value under the new distribution obtained from (1) by multiplying  $e_L$  and  $e_H$  by  $k$ , we define

$$\Delta^{\text{type1}}[X] \equiv \lim_{k \rightarrow 1} \frac{X(ke_L, ke_H) - X(e_L, e_H)}{k - 1}. \quad (\text{A12})$$

Analogously, denoting by  $X(e_H)$  the value of  $X$  under the initial distribution (1) and by  $X(ke_H)$  the value under the new distribution obtained from (1) by multiplying  $e_H$  by  $k$ , we have

$$\Delta^{\text{type2}}[X] \equiv \lim_{k \rightarrow 1} \frac{X(ke_H) - X(e_H)}{k - 1}, \quad (\text{A13})$$

and replacing  $e_H$  by  $e_L$  in the above description yields

$$\Delta^{\text{type3}}[X] \equiv \lim_{k \rightarrow 1} \frac{X(ke_L) - X(e_L)}{k - 1}. \quad (\text{A14})$$

To calculate the Gini index, we first compute the share  $r(q)$  of the overall wealth (3) that belongs to the fraction  $q \in [0, 1]$  of investors in the lower end of wealth distribution. Recalling that investors on the interval  $[0, 1]$  are sorted by wealth, the bottom  $q$  of investors lie on  $[1 - q, 1]$ , and so using (1) and (3)  $r(q)$  is computed as

$$r(q) = \frac{\int_{1-q}^1 e_H / (1 + i(e_H - e_L)/e_L) di}{\bar{e}} = \frac{\ln e_H - \ln(e_L + (e_H - e_L)(1 - q))}{\ln(e_H/e_L)}. \quad (\text{A15})$$

By definition, the Gini index is

$$G \equiv 1 - 2 \int_0^1 r(q) dq, \quad (\text{A16})$$

and so from (A15)

$$G = 1 - 2 \int_0^1 \frac{\ln e_H - \ln(e_L + (e_H - e_L)(1 - q))}{\ln(e_H/e_L)} dq. \quad (\text{A17})$$

Using the following relation

$$\int \ln(Ax + B) = \frac{(Ax + B) \ln(Ax + B) - (Ax + B)}{A}, \quad (\text{A18})$$

we compute the integral in (A17), which after some transformations leads to (15).

Turning to how  $G$  is affected by wealth growth, the result that  $\Delta^{\text{type1}}[G] = 0$  is seen by eyeballing expression (15)—looking at the second term on the right-hand side, we see that multiplying both  $e_L$  and  $e_H$  by  $k$  implies that both the numerator and the denominator are multiplied by  $k$ , and so, from definition (A12), there is no effect on  $G$ . Establishing the

two other properties is more involved, and we now derive only the property  $\Delta^{\text{type}2}[G] > 0$ ; establishing  $\Delta^{\text{type}3}[G] < 0$  is analogous. Differentiating  $G$  given by (15) with respect to  $e_H$  and rearranging, we get

$$\Delta^{\text{type}2}[G] = -2[(e_L - e_H)(-e_L + e_H + e_L \ln e_L - e_L \ln e_H) + e_L(e_L - e_H - e_H \ln e_L + e_H \ln e_H) \ln(e_H/e_L)] / [(e_L - e_H)^2 e_H \ln(e_H/e_L)^2] \quad (\text{A19})$$

Because we are interested in the sign of the derivative, we drop the denominator in (A19) which is always positive, and get

$$\begin{aligned} \text{sgn} \left[ \frac{\partial G}{\partial e_H} \right] &= -\text{sgn}[(e_L - e_H)(-e_L + e_H + e_L \ln e_L - e_L \ln e_H) \\ &+ e_L(e_L - e_H - e_H \ln e_L + e_H \ln e_H) \ln(e_H/e_L)] = \text{sgn}[(e_H - e_L)^2 - e_H e_L (\ln(e_H/e_L))^2] \end{aligned} \quad (\text{A20})$$

Making a change of variable  $z \equiv e_H/e_L$  and plugging this in (A20), we obtain

$$\text{sgn}[(e_H - e_L)^2 - e_H e_L (\ln(e_H/e_L))^2] = \text{sgn}[(z - 1)^2 - z(\ln z)^2] = \text{sgn}[(z - 1)/\sqrt{z} - \ln z], \quad (\text{A21})$$

where we look only at the region  $z > 1$  because  $e_H > e_L$ . Hence, proving that  $\frac{\partial G}{\partial e_H}$  is positive is equivalent to proving that  $f(z) \equiv (z - 1)/\sqrt{z} - \ln z$  is positive for  $z > 1$ . To see this, note that  $f(1) = 0$  and

$$f'(z) = \frac{1}{2\sqrt{z}} + \frac{1}{2z^{3/2}} - \frac{1}{z} = \frac{z - 2\sqrt{z} + 1}{2z^{3/2}} = \frac{(\sqrt{z} - 1)^2}{2z^{3/2}} > 0,$$

from which it follows that  $f(z) > 0$  for  $z > 1$ .

*Q.E.D.*

**Proof of Proposition 3.** Substituting  $e_i$  by  $e^*$  and  $i$  by  $i^*$  in (1), we get

$$e^* = \frac{e_H}{1 + i^*(e_H - e_L)/e_L}, \quad (\text{A22})$$

plugging which in (12) and rearranging yields (16). We now consider in turn the three types of wealth grows and prove the inequalities stated in Proposition 3.

**Type 1 wealth growth.** Using definition (A12), we replace  $e_H$  and  $e_L$  in (16) by  $ke_H$  and  $ke_L$ , differentiate with respect to  $k$  and evaluate the derivative at  $k = 1$ . Making the required replacements in (16) and transferring all terms to the left-hand side, we obtain an equation of the form  $F(i^*, k) = 0$ , where  $F(\cdot)$  is

$$F(i^*, k) \equiv (i^* + e_L/(e_H - e_L)) \ln(1 + i^*(e_H - e_L)/e_L) - i^* - k\bar{e}a\gamma\sigma^2. \quad (\text{A23})$$

This equation defines  $i^*$  as an implicit function of  $k$ , and differentiating this implicit function yields

$$\Delta^{\text{type1}}[i^*] = \frac{di^*}{dk} = -\frac{\partial F/\partial k}{\partial F/\partial i^*} = \frac{\bar{e}a\gamma\sigma^2}{\ln\left(1 + i^*\frac{e_H - e_L}{e_L}\right) + \frac{(i^* + \frac{e_L}{e_H - e_L})(e_H - e_L)}{e_L(1 + i^*(e_H - e_L)/e_L)} - 1}, \quad (\text{A24})$$

which after cancelations leads to

$$\frac{di^*}{dk} = \frac{\bar{e}\gamma\sigma^2}{\ln(1 + i^*(e_H - e_L)/e_L)} > 0.$$

Analogously, we replace  $e_H$  and  $e_L$  in (12) by  $ke_H$  and  $ke_L$ , transfer all terms to the left-hand side, and define function  $M(\cdot)$  as

$$M(e^*, k) \equiv \frac{ke_H}{e^*} \ln \frac{ke_H}{e^*} - \frac{ke_H}{e^*} - a\gamma\sigma^2 ke_H \ln \frac{e_H}{e_L} + 1, \quad (\text{A25})$$

and so

$$\frac{\partial e^*}{\partial k} = -\frac{\partial M/\partial k}{\partial M/\partial e^*} = \frac{\frac{e_H}{e^*}(\ln \frac{ke_H}{e^*} - 1) + \frac{ke_H}{e^*} \frac{1}{k} - a\gamma\sigma^2 e_H \ln \frac{e_H}{e_L}}{\frac{ke_H}{(e^*)^2}(\ln \frac{ke_H}{e^*} - 1) + \frac{ke_H}{e^*} \frac{1}{e^*}} = \frac{\frac{ke_H}{e^*} - 1}{\frac{ke_H}{(e^*)^2} \ln \frac{ke_H}{e^*}}, \quad (\text{A26})$$

where the last equality is obtained by computing  $a\gamma\sigma^2 e_H \ln \frac{e_H}{e_L}$  from (A25) (recall that  $M(\cdot) = 0$ ), and substituting the result in the numerator of the last but one term.

Differentiating (14) and using (A26), we get

$$\Delta^{\text{type1}}[\mu - p] = -\frac{1}{a(e^*)^2} \left. \frac{\partial e^*}{\partial k} \right|_{k=1} = -\frac{1}{a(e^*)^2} \frac{\frac{e_H}{e^*} - 1}{\frac{e_H}{(e^*)^2} \ln \frac{e_H}{e^*}} < 0,$$

where the last inequality follows from the fact that in the limited participation case  $e^* < e_H$ .

**Type 2 wealth growth.** In line with definition (A13), we set  $k = 1$  in (A23) and use the resulting equation to find the derivative of an implicit function  $i^*(e_H)$ , we get

$$\frac{\partial i^*}{\partial e_H} = -\frac{\partial F/\partial e_H}{\partial F/\partial e^*} = \frac{e_H(e_L a\gamma\sigma^2 - i^*) + e_L \left( -e_L a\gamma\sigma^2 \left( 1 + \ln \frac{e_H}{e_L} \right) + \ln \left[ 1 + i^* \frac{e_H - e_L}{e_L} \right] + i^* \right)}{\ln \left[ 1 + i^* \frac{e_H - e_L}{e_L} \right] (e_L - e_H)^2}. \quad (\text{A27})$$

We are interested in the sign of the above derivative, and so we drop the denominator which is positive, which yields after some transformations

$$\text{sgn} \left[ \frac{\partial i^*}{\partial e_H} \right] = \text{sgn} \left[ e_L a\gamma\sigma^2 \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) - e_H i^* + e_L \ln \left( 1 + i^* \frac{e_H - e_L}{e_L} \right) + e_L i^* \right] \quad (\text{A28})$$

Expressing  $a\gamma\sigma^2$  from (16) and substituting the resulting expression into (A28), we get that (A28) is equivalent to

$$\text{sgn} \left[ e_L \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) \left( \left( i^* + \frac{e_L}{e_H - e_L} \right) \ln \left( 1 + i^* \frac{e_H - e_L}{e_L} \right) - i^* \right) - \bar{e} e_H i^* + \bar{e} e_L \ln \left( 1 + i^* \frac{e_H - e_L}{e_L} \right) + \bar{e} e_L i^* \right]. \quad (\text{A29})$$

Denoting by  $h(i^*)$  the argument of the sign function in (A29), it is easy to see that  $h(0) = 0$ .

Moreover, we have

$$\begin{aligned}
h(1) &= e_L \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) \left( \frac{e_H}{e_H - e_L} \ln \frac{e_H}{e_L} - 1 \right) - \bar{e}e_H + \bar{e}e_L \ln \frac{e_H}{e_L} + \bar{e}e_L \\
&= e_L e_H \ln \frac{e_H}{e_L} - e_L(e_H - e_L) - \bar{e}e_L \ln \frac{e_H}{e_L} + e_L^2 \ln \frac{e_H}{e_L} - \bar{e}e_H + \bar{e}e_L \ln \frac{e_H}{e_L} + \bar{e}e_L \\
&= e_L e_H \ln \frac{e_H}{e_L} - e_L(e_H - e_L) + e_L^2 \ln \frac{e_H}{e_L} - \bar{e}e_H + \bar{e}e_L = -e_L(e_H - e_L) + e_L^2 \ln \frac{e_H}{e_L},
\end{aligned} \tag{A30}$$

where in (A30) we make use of the expression for total endowment  $\bar{e}$  given in (3). Taking a well-known inequality  $a > 1 + \ln a$ ,  $a > 1$ , and setting  $a = e_H/e_L$ , we get that the last expression in (A30) is negative, and so  $h(1) < 0$ .

Differentiating (A29) yields

$$h'(i^*) = \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) \ln \left( 1 + i^* \frac{e_H - e_L}{e_L} \right) e_L - \bar{e}(e_H - e_L) + \frac{e_H - e_L}{1 + i^* \frac{e_H - e_L}{e_L}} \bar{e}, \tag{A31}$$

from which we get  $h'(0) = 0$ . Differentiating (A31), we get

$$h''(i^*) = \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) \frac{e_H - e_L}{\left( 1 + i^* \frac{e_H - e_L}{e_L} \right)} - \frac{(e_H - e_L)^2/e_L}{\left( 1 + i^* \frac{e_H - e_L}{e_L} \right)^2} \bar{e}, \tag{A32}$$

and so

$$\begin{aligned}
h''(0) &= \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} \right) (e_H - e_L) - \bar{e}(e_H - e_L)^2/e_L = \\
&= (e_H - e_L) \left( e_H - e_L - e_L \ln \frac{e_H}{e_L} - e_H \ln \frac{e_H}{e_L} \right) = \frac{e_H - e_L}{e_L} \left( \frac{e_H}{e_L} - \frac{e_H}{e_L} \left( \ln \frac{e_H}{e_L} + 1 \right) - 1 \right) < 0.
\end{aligned}$$

Equating (A32) to zero, it is easy to see that there is a unique solution  $\tilde{i}^*$  to this equation, meaning that  $h(\cdot)$  has only one inflection point  $\tilde{i}^*$ . We now consider two cases. If  $\tilde{i}^*$  is outside the interval  $[0,1]$ , then  $h(\cdot)$  is a decreasing function on  $(0,1)$  because, as established above,

$h(0) = 0, h'(0) = 0, h''(0) < 0, h(1) < 0$ . If, on the other hand,  $\tilde{i}^* \in (0, 1)$ , then it is still the case that  $h(\cdot)$ , while being convex in the region  $[\tilde{i}^*, 1]$ , does not become positive, because if it did, there had to be another inflection point  $\hat{i}^* > \tilde{i}^*$  in order to have  $h(1) < 0$ , which is a contradiction. Hence,  $h(i^*) < 0$  for  $i^* \in (0, 1)$ , and so the sign of the derivative in (A27) is negative. Hence,  $\Delta^{\text{type2}}[i^*] < 0$

Setting  $k = 1$  in (A25) and using the resulting equation to compute the derivative of an implicit function  $e^*(e_H)$ , we get

$$\frac{\partial e^*}{\partial e_H} = -\frac{\partial M/\partial e_H}{\partial M/\partial e^*} = \frac{\ln(e_H/e^*)/e^* - a\gamma\sigma^2(1 + \ln(e_H/e_L))}{e_H \ln(e_H/e^*)}(e^*)^2, \quad (\text{A33})$$

substituting which into

$$\frac{\partial(\mu - p)}{\partial e} = -\frac{1}{a(e^*)^2} \frac{\partial e^*}{\partial e_H},$$

we get

$$\text{sgn} \left[ \frac{\partial(\mu - p)}{\partial e_H} \right] = \text{sgn} \left[ a\gamma\sigma^2 e^* \left( 1 + \ln \frac{e_H}{e_L} \right) - \ln \frac{e_H}{e^*} \right]. \quad (\text{A34})$$

Expressing  $a\gamma\sigma^2$  from (12) and substituting the resulting expression into (A34), we get that (A34) is equivalent to

$$\text{sgn} \left[ \frac{(e_H \ln \frac{e_H}{e^*} - e_H + e^*)(1 + \ln \frac{e_H}{e_L})}{e_H \ln \frac{e_H}{e_L}} - \ln \frac{e_H}{e^*} \right]. \quad (\text{A35})$$

Denote by  $g(e^*)$  the argument of the sign function in (A35). It is easy to see that  $g(e_H) = 0$ .

Next, we find  $g(e_L)$ :

$$g(e_L) = \frac{(e_H \ln \frac{e_H}{e_L} - e_H + e_L)(1 + \ln \frac{e_H}{e_L})}{e_H \ln \frac{e_H}{e_L}} - \ln \frac{e_H}{e_L} = \frac{e_L + e_L \ln \frac{e_H}{e_L} - e_H}{e_H \ln \frac{e_H}{e_L}}. \quad (\text{A36})$$

We again use the inequality  $a > 1 + \ln a, a > 1$  in which we set  $a = e_H/e_L$ , implying that the numerator in (A36) is negative, and so  $g(e_L) < 0$ . Finally, we compute the second derivative



of  $g(e^*)$ :

$$g''(e^*) = \frac{1}{(e^*)^2 \ln \frac{e_H}{e_L}} > 0.$$

To sum up,  $g(e^*)$  is a convex function with  $g(e_L) < 0$  and  $g(e_H) = 0$ . Hence,  $g(e^*) < 0$  for  $e^* \in [e_L, e_H]$ , implying that the sign of the expressions in (A34) is negative, and so  $\Delta^{\text{type2}}[\mu - p] < 0$ .

**Type 3 wealth growth.** The analysis here is very similar to that for type 2 wealth growth, so we only present some key expressions. Equations (A23) and (A25) define implicitly  $i^*(e_L)$  and  $e^*(e_L)$ , respectively. Differentiating these implicit functions, we get

$$\frac{\partial e^*}{\partial e_L} = \frac{a\gamma\sigma^2 (e^*)^2}{\ln(e_H/e^*)e_L}, \tag{A37}$$

$$\frac{\partial i^*}{\partial e_L} = \frac{e_H (a\gamma\sigma^2 e_L^2 + e_H i^* - e_L (\ln(1 + (e_H/e_L - 1)i^*) + (a\gamma\sigma^2 - a\gamma\sigma^2 \ln(e_H/e_L)) e_H + i^*))}{\ln(1 + (e_H/e_L - 1)i^*) (e_H - e_L)^2 e_L}. \tag{A38}$$

The derivative in (A37) is positive, and hence from (14) we obtain that  $\Delta^{\text{type3}}[\mu - p] < 0$ . Establishing the sign of (A38) is more cumbersome, but following the analogous approach to that used for (A27) we obtain  $\Delta^{\text{type3}}[i^*] > 0$ . *Q.E.D.*

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